An EOQ Model Dealing with Weibull Deterioration with Shortages, Ramp Type Demand Rate and Unit Production Cost Incorporating the Effect of Inflation

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Abstract: As the long arm of the grinding, deep financial crisis continues to haunt the global economy and the assumption that the goods in inventory always preserve their physical characteristic are not true in general, the effects of inflation and deterioration cannot be oblivious to an inventory system. In today's market structure, partial backlogged shortages are a more practical important assumption for better business performance. Therefore, bearing in mind these facts, we have developed an order level inventory system for deterioration with shortages and demand rate as a ramp type function of time by exploring two different cases when the demand rate is stabilized after and before the production stopping time end unit production cost is inversely proportional to the demand rate. Optimal costs are determined by two different cases incorporating the effect of inflation.

Keywords: Ramp Type Demand, Weibull Deterioration, Unit Production Cost, With Shortage,inflation.

Introduction

Traditionally items kept as inventory are tacitly assumed to have an infinite lifespan or presumed to be perfect throughout the business cycle. In today's competitive business world, it is of the unrealistic assumption that all produced items are of good quality. But for managing inventory in a realistic scenario, many products impair in quality due to changing technological trends, it would be unethical if the phenomenon deterioration are not considered. Deterioration of items is a frequent and natural phenomenon which cannot be ignored. In realistic scenario the life cycle of seasonal product, fruits, food items, electric component, volatile liquid, food etc are short and finite usually can undergo deterioration. Thus the item may not serve the purpose after a period of time and will have to be discarded as it cannot be used to satisfy the future demand of customers. The term deterioration means spoilage, vaporization and obsolescence, loss of character or value in a product along time. At first Wagner and Whitin (1958) dealt with an inventory model for deteriorating items at the end of the shortage period. Researchers have been progressively modifying the existing models by using the deterioration function of various types; it can be constant type or dependent on time. .In our proposed model we have considered the Weibull distribution as the function of deterioration. In past few decades it is observed by Berrotoni (1962) that both the leakage failure for the dry batteries and the life expectancy of ethical drugs could be expressed in Weibull distribution. Covert and Philip (1973) has influenced by the work of Berrotoni (1962) to develop an inventory model for deteriorating items with variable rate of deterioration .They have explained two parameter Weibull distribution to contemplate deterioration as distribution of time. Misra (1975) proposed an inventory model with two parameter Weibull distribution and finite rate of replenishment. The research has been summarized in different survey papers Goyal and Giri (2001), Raafat (1991), Ruxian et al. (2010), Goyal et al. (2013).

For long, inventory models have discussed with the case where demand is either a constant or a monotonic function. Almost the time varying demand patterns considered in most of the papers where assum es demand rate to be either increasing or decreasing continuously. But in practice, it is not possible. In real world, it stabilizes at the mature stage of the product life cycle once the product has been accepted in the market. The kind of stabilization has been termed as "ramp type". The ramp type demand is commonly seen when some fresh fruits are brought to the market. Hill (1995) first time considered increases linearly at the beginning and then after maturation it becomes a constant, a stable stage till the end of the inventory cycle. Deng at al. (2007) developed a note on the inventory models the deteriorating items with ramp-type demand rate by exploring two cases where the time point occurs before and after the point where the demand is stabilized. Skouri et al. (2011) studied with ramp type demand rate and time dependent deterioration rate with unit production cost and shortages. This type of demand patterns examined by Hariga (1995), Wu and Ouyang (2000), Yang et al. (2001), Girl et al. (2003), Manna and Chaudhuri (2006), Panda et al. (2008), Chen et al.(2006) etc.

Apart from the above mentioned facts due to the globalization of market inflation cannot be shrugged off while doing the inventory models as the market fluctuates from time to time. So inflation is a crucial attribute of today's esoteric economy. However the most common economic meaning of inflation is: reduction in the value of money. Buzacott (1975) developed the model by considering inflationary effect assuming constant inflation rate. Thereafter a great deal of research efforts have been developed or Liao et al. (2000), Yang et al. (2001), Sarkar et al. (2011). Roy and Chaudhuri (2011) dealt with an EOQ model with ramp type demand under finite time horizon and inflation.
Moreover in today's customer are often fickle and increasingly less loyal, some customers would like to wait during the shortage period, but others would not. The customer's demand for the item is lost in the lost sales case and an assuredly filled by a competitor, which can be manifestly as the loss of profit on the sales. Resultantly the opportunity cost resulted from lost sales should be deliberated in the model. Some studies have mutated inventory policies by considered the partial backlogging rate and some assumed to be completely backlogged. Two types of backlogging accumulated such as constant type and time dependent partial backlogging rate dependent on the wailing time up to the next replenishment have been studied extensively by many researches such as Abad (1996), Chang and Dye (1999), Wang (2002), Wu et al. (2006), Singh and Singh (2007), Dye et al.(2007), Singh et al.(2008), Sicilia et al. (2009). However in market structure another important factor is shortages which no retailer would prefer, and in practice are partially backlogged and partially lost.
Present work is the extension of Manna et al(in press) without shortage model where (a) the demand rate is stabilized into two different cases i.e. after the production stopping time and before the production stopping time and (b) Deterioration rate is constant .In the proposed model we at first have the demand rate which is realistic as any new brands product launch in the market the demand rate linearly depends on time and is stabilized after the production stopping the time and before the time when inventory level reaches zero. Secondly we can ignore the effect of deterioration over time as it is a natural phenomenon so we have taken the deterioration as Weibull distribution. In our model we have taken costs are considered under the effects of learning and inflation.
Further, the paper is validated with the help of a numerical example. The model has explored the effects of deterioration, inflation and backlogging parameter and finds the optimal costs in two different cases with the effect of learning and inflation.

Assumptions and Notations

The following notations and assumptions are considered to develop the inventory model

Notations

K- Unit Production cost (units /unit time)
c_1 − Holding cost per unit per unit of time
c_3 − Deterioration cost per unit per unit of time
c_4 − Shortage cost per unit per unit of time
c_5 − Lost sale cost per unit per unit of time
X – Total average cost for a production cycle
r- Inflationary rate
δ – Backlogging rate

Assumptions

(1) Demand rate in ramp type function of time, i.e. demand rate R= f (t) is assumed to be a ramp type function of time f(t)= D_0[t-(t-μ) H(t-μ)] , $D_0 > 0$ and H(t) is a Heaviside's function:
$$H(t-\mu) = \begin{cases} 1 & if\ t \geq \mu \\ 0 & if\ t < \mu \end{cases}$$
(2) Deterioration varies unit time and it is function of two parameter Weibull distribution of the time, i.e. $\alpha\beta t^{\beta-1}$, $0 < \alpha < 1$, $\beta \geq 1$, where t denote time of deterioration .
(3) Lead time is zero.
(4) Inflation is considered.
(5) Shortage are Allowed and partially backlogged.
(6) K= γ f (t) is the production rate where γ (> 1) is a constant.

The unit production cost $v = \alpha_1 R^{-s}$ where $\alpha_1 > 0, s > 0$ and s $\neq$ 2.
α_1 is obviously positive since v and R are both non-negative. Also higher demands result in lower unit cost of production. This implies that v and R are inversely related and hence, must be non-negative i.e. positive.
Now,
$$\frac{dV}{dR} = -\alpha_1 s R^{-(s+1)} < 0.$$

$$\frac{d^2V}{dR^2} = \alpha_1 s(s+1)R^{-(s+2)} > 0.$$

Thus, marginal unit cost of production is an increasing function of R. These results imply that, as the demand rate increases, the unit cost of production decreases at an increasing rate. Due to this reason, the manufacture is encouraged to produce more as the demand for the item increases. The necessity of restriction $s \neq 2$ arises from the nature of the solution of the problem.

Mathematical Formulation of the Model

Case 1 ($\mu \le t_1 \le t_2$)

The stock level initially is zero. Production starts just after t=0. When the stock attains a level q at time t= t_1, then the production stops at that time. The time point μ occurs before the point t=t_1, where demand is stabilized after that the inventory level diminishes due to both demand and deterioration ultimately falls to zero at time t = t_2. After time t_2 shortages occurs at t=T, which are partially backlogged and partially lost. Then, the cycle repeats.

Let Q(t) be the inventory level of the system at any time t($0 \le t \le t_2$). The differential equations governing the system in the interval [0,t_2]are given by

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = K - F(t) \qquad 0 \le t \le \mu \qquad (1)$$

with the condition Q(0)=0

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = K - F(t) \qquad \mu \le t \le t_1 \qquad (2)$$

with the condition $Q(t_1) = q$

$$\frac{dQ(t)}{dt} + \alpha \beta t^{\beta-1} Q(t) = -F(t) \qquad t_1 \le t \le t_2 \qquad (3)$$

with the condition $Q(t_1) = q, Q(t_2)=0$

$$\frac{dQ(t)}{dt} = -e^{-\delta(T-t_2)} F(t) \qquad t_2 \le t \le T \qquad (4)$$

with the condition $Q(t_2)=0$

Using ramp type function F(t), equation (1),(2),(3),(4) become respectively

$$\frac{dQ(t)}{dt} + \alpha \beta t^{\beta-1} Q(t) = (\gamma-1)D_0 t \qquad 0 \le t \le \mu \qquad (5)$$

with the condition Q(0) = 0

$$\frac{dQ(t)}{dt} + \alpha \beta t^{\beta-1} Q(t) = (\gamma-1)D_0 \mu \qquad \mu \le t \le t_1 \qquad (6)$$

with the condition $Q(t_1) = q$

$$\frac{dQ(t)}{dt} + \alpha \beta t^{\beta-1} Q(t) = D_0 \mu \qquad t_1 \le t \le t_2 \qquad (7)$$

With the conditions $Q(t_1)$= q, $Q(t_2)$= 0,

$$\frac{dQ(t)}{dt} = -e^{-\delta(T-t_2)} D_0 \mu \qquad t_2 \le t \le T \qquad (8)$$

with the condition $Q(t_2)$=0

(5),(6),(7),(8) are first order linear differential equations

For the solution of equation (5) we get

$$Q(t)e^{\alpha t^\beta} = (\gamma-1)\int D_0 t e^{\alpha t^\beta} + C$$

$$= (\gamma-1)D_0 \left[\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + --\right] + C \qquad (9)$$

By using the condition Q (0) = 0 (10)

$$Q(t) = (\gamma-1)D_0 e^{-\alpha t^\beta}\left[\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + --\right], 0 \le t \le \mu \qquad (11)$$

for the solution of equation (6) we have

$$\int_\mu^t d[e^{\alpha t^\beta} Q(t)] = (\gamma-1)\, D_0\, \mu \int_\mu^t e^{\alpha t^\beta} dt$$

$$e^{\alpha t^\beta} Q(t) - e^{\alpha \mu^\beta} Q(\mu) = (\gamma-1)\, D_0\, \mu \int_\mu^t e^{\alpha t^\beta} dt = (\gamma-1) D_0\, \mu \int_\mu^t 1 + \alpha t^\beta +$$

$$e^{\alpha t^\beta} Q(t) - e^{\alpha \mu^\beta} Q(\mu) = (\gamma-1)\, D_0\, \mu\left[t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)} + ----\right]$$

$$e^{\alpha t^\beta} Q(t) = (\gamma-1)\, D_0\, \mu\left[\frac{\mu}{2} + \frac{\alpha \mu^{\beta+1}}{\beta+2} + \frac{\alpha^2 \mu^{2\beta+1}}{2(2\beta+2)} \pm --\right] + (\gamma-1) D_0 \mu \left[t - \mu + \frac{\alpha}{\beta+1}\left(\frac{t^{\beta+1}}{\mu^{\beta+1}}\right) + \frac{\alpha^2}{2(2\beta+1)}\left(t^{2\beta+1} - \mu^{2\beta+1}\right) + \right]$$

$$= (\gamma-1) D_0 \mu e^{-\alpha t^\beta}\left[t - \frac{\mu}{2} + \frac{\alpha t^{\beta+1}}{(\beta+1)} + \frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha \mu^{\beta+1}}{(\beta+1)(\beta+2)} - \frac{\alpha^2 \mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)}\right], \mu \le t \le t_1 \qquad (12)$$

The solution of equation (7) is given by

Q(t) $e^{\alpha t^{\beta}} = -D_0\mu \int e^{\alpha t^{\beta}} dt + C$

$= - D_0\mu \int [1 + \alpha t^{\beta} + \frac{\alpha^2 t^{2\beta}}{2} + \quad] + C$

Q(t) $e^{\alpha t^{\beta}} = -Do\mu \left(t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)} + \quad \right) + C$

Putting $Q(t)_1$ = q we get

$$\mathrm{q}e^{\alpha t_1 \beta} = -Do\mu \left(t_1 + \frac{\alpha t_1 \beta + 1}{\beta + 1} + \frac{\alpha^2 t_1^{2\beta+1}}{2(2\beta + 1)} + \qquad \right) + C$$

C=q$e^{\alpha t_1^{\beta}} + Do\mu \left(t_1 + \frac{\alpha t_1 \beta+1}{\beta+1} + \frac{\alpha^2 t_1^{2\beta+1}}{2(2\beta+1)} + \qquad \right)$ (13)

Using initial condition Q(t_2)=0 in equation (13) we have,

$qe^{\alpha t_1^{\beta}} = D_0 \mu$ ($t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_2{}^{2\beta+1}}{2(2\beta+1)} + ---) - D_0\mu\,(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_1{}^{2\beta+1}}{2(2\beta+1)} + \qquad)$

q= $D_0\ \mu e^{-\alpha t_1^{\beta}}\,(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_2{}^{2\beta+1}}{2(2\beta+1)} + \quad) - D_0\mu\, e^{-\alpha t_1^{\beta}}\,(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_1{}^{2\beta+1}}{2(2\beta+1)} + --)$

Substitute q in equation (13) the solution of equation (7) is

$e^{\alpha t^{\beta}}Q(t) = -D_0\, \mu \left(t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)} + ---\right) + D_0\mu \left(t_{2+} \frac{\alpha t_2{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_2{}^{2\beta+1}}{2(2\beta+1)} + --\right) - D_0\mu\left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_1{}^{2\beta+1}}{2(2\beta+1)} + --\right) + D_0\mu\,(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_1{}^{2\beta+1}}{2(2\beta+1)} + --)$

$Q\ (t) = D_0\ \mu\, e^{-\alpha t^{\beta}}[(\,t_2 - t) + \frac{\alpha}{\beta+1}(\,t_2{}^{\beta+1} - t^{\beta+1}) + \frac{\alpha^2}{2(2\beta+1)}\left(\,t_2{}^{2\beta+1} - t^{2\beta+1}\right) + --\,]$

$t_1 \leq t \leq t_2$ (15)

The solution of equation (8) is

$\frac{dQ(t)}{dt} = -D_0\mu\, e^{-\delta(T-t_2)}$ $\qquad t_2 \leq t \leq T$

with boundary condition $Q(t_2) = 0$

$Q\ (t) = -D_0\ \mu\ \int [1 - \delta(T - t_2)]\ dt + c$

$Q\ (t) = -D_0\ \mu\ [t - \delta(T - t_2)t] + c$

By using $Q(t_2)$ =0, we get

$Q(t) = D_0\ \mu[(t_2 - t) - \delta(T - t_2)(t_2 - t)]$ (16)

Shortage cost over the period [0,T] is defined as

$\int_{t_2}^{t} \theta(t)dt = -\int_{t_2}^{T} D_0\ \mu[\{t_2 - t) - \delta[T - t_2](t_2 - t)]dt\, e^{-rt}$

$= -D_0\ \mu \int_{t_2}^{T}(1 - rt)[(t_2 - t) - \delta(T - t_2)(t_2 - t)]dt$

$= -D_0\ \mu \left[\left\{Tt_2 - \frac{t_2^2}{2} - \frac{T^2}{2} - \delta(T - t_2)\left(t_2 T - \frac{t_2^2}{2} - \frac{T^2}{2}\right)\right\} - \left\{r\left(\frac{T^2}{2} - \frac{t_2^2}{2}\right)t_2 - r\left(\frac{T^3}{3} - \frac{t_2^3}{3}\right) - \delta r(T - t_2)\left(\frac{t_2 T^2}{2} - \frac{t_2^3}{2} - \frac{T^3}{3} + \frac{t_2^3}{3}\right)\right\}\right]$

$= -D_0\ \mu \left[\left\{Tt_2 - \frac{t_2^2}{2} - \frac{T^2}{2} - \delta\left(\frac{3t_2 T^2}{2} - \frac{3t_2^2 T}{2} - \frac{T^2}{2} + \frac{t_2^3}{2}\right) - \left\{r\frac{T^2 t_2}{2} + r\frac{t_2^3}{2} + r\frac{T^3}{3} + \delta r\ (T - t_2)\left(\frac{t_2 T^2}{2} - \frac{t_2^3}{6} - \frac{T^3}{3}\right)\right\}\right]$ `

(17)

Lost sale cost per cycle is

LS= $D_0\ \mu \int_{t_2}^{T}(1 - e^{-\delta(T-t_2)})\ dt$

LS= $D_0\ \mu \int_{t_2}^{T} \delta(T - t_2)(t_2 - t)\ dt$

Lost sale cost over the period [0,T] is

LS= $D_0\ \ \mu \int_{t_2}^{T} \delta(T - t_2)(t_2 - t)e^{-rt}\ dt$

LS= $D_0\ \mu\delta \int_{t_2}^{T}(T - t_2)(t_2 - t)\ (1 - rt)d$

LS= $D_0\ \mu\delta \int_{t_2}^{T}(Tt_2 - Tt - t_2^2 + t_2 t - rTt_2 t + rTt^2 + rt_2^2 t - rt_2 t^2)\ dt$

LS= $D_0\ \mu\delta \left[Tt_2(T - t_2) - T\ \left(\frac{T^2}{2} - \frac{t_2^2}{2}\right) - t_2^2(T - t_2) + t_2\left(\frac{T^2}{2} - \frac{t_2^2}{2}\right) - rTt_2\left(\frac{T^2}{2} - \frac{t_2^2}{2}\right) + rT\ \left(\frac{T^3}{3} - \frac{t_2^3}{3}\right) + rt_2^2\left(\frac{T^2}{2} - \frac{t_2^2}{2}\right) - rt_2\left(\frac{T^3}{3} - \frac{t_2^3}{3}\right)\right]$

LS= $D_0\ \mu\delta \left[\frac{3T^2 t_2}{2} - \frac{3t_2^2 T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2} - \frac{5rt_2 T^3}{6} - \frac{rt_2^4}{6} + \frac{rTt_2^3}{6} + \frac{rT^4}{3} + \frac{rt_2^2 T^2}{2}\right]$ (18)

The total inventory over the period [0, t_2] is

$$\int_0^{t_2} Q\,(t)dte^{-rt} = \int_0^{\mu} Q\,(t)e^{-rt}dt + \int_{\mu}^{t_1} Q\,(t)e^{-rt}dt + \int_{t_1}^{0} Q\,(t)e^{-rt}dt$$

$$\int_0^{\mu} Q(t)e^{-rt}dt = \int_0^{\mu} (\gamma-1)D_0e^{-\alpha t^{\beta}}[\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + --]e^{-rt}dt$$

$$= (\gamma-1)D_0\int_0^{\mu}\left(\frac{t^2}{2} - \frac{\alpha\beta t^{\beta+2}}{2(\beta+2)} - \frac{\alpha^2\,(3\beta+2)t^{2\beta+2}}{2(\beta+2)(2\beta+2)}\right)(1-rt)dt$$

$$=(\gamma\text{-}1)\,D_0\ \int_0^{\mu}(\frac{t^2}{2} - \frac{\alpha\beta t^{\beta+2}}{2(\beta+2)} - \frac{\alpha^2(3\beta+2)t^{2\beta+2}}{2(\beta+2)(2\beta+2)} - \frac{rt^3}{2} + \frac{\alpha r\beta t^{\beta+3}}{2(\beta+2)} - \frac{\alpha^2(3\beta+2)t^{2\beta+3}}{2(\beta+2)(2\beta+2)})\ \mathrm{dt}$$

$$=(\gamma\text{-}1)\,D_0\ [\frac{t^3}{6} - \frac{\alpha\beta t^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha^2(3\beta+2)t^{2\beta+3}}{2(\beta+2)(2\beta+2)(2\beta+3)} - \frac{rt^4}{8} + \frac{r\alpha\beta t^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{r\alpha^2(3\beta+2)t^{2\beta+4}}{2(\beta+2)(2\beta+2)(2\beta+4)}]_0^{\mu}$$

$$= (\gamma\text{-}1)\,D_0\ \ [\frac{\mu^3}{6} - \frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(\beta+2)(2\beta+2)(2\beta+3)} - \frac{r\mu^2}{8} + \frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{2(\beta+2)(2\beta+2)(2\beta+4)}]$$

$$\int_{\mu}^{t_1} Q(t)e^{-rt}dt = D_0\,\mu(\gamma-1)\int_{\mu}^{t_1}[t - \frac{\mu}{2} + \frac{\alpha t^{\beta+1}}{(\beta+1)} + \frac{\alpha^2t^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha\mu^{\beta+1}}{(\beta+1)(\beta+2)} - \frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)} - \alpha t^{\beta+1} + \frac{\alpha\mu t^{\beta}}{2} - \frac{\alpha^2t^{2\beta+1}}{(\beta+1)} + \frac{\alpha^2t^{\beta}\mu^{\beta+1}}{(\beta+1)(\beta+2)}]\,e^{-rt}\ dt$$

$$= D_0\mu\,(\gamma-1)\int_{\mu}^{t_1} t - \frac{\mu}{2} + \frac{\alpha t^{\beta+1}}{(\beta+1)} + \frac{\alpha^2t^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha\mu^{\beta+1}}{(\beta+1)(\beta+2)} - \frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)} - \alpha t^{\beta+1} + \frac{\alpha\mu t^{\beta}}{2} - \frac{\alpha^2t^{2\beta+1}}{(\beta+1)} + \frac{\alpha^2t^{\beta}\mu^{\beta+1}}{(\beta+1)(\beta+2)} - rt^2 + rt\frac{\mu}{2} - \frac{\alpha rt^{\beta+2}}{(\beta+1)} - \frac{r\alpha^2t^{2\beta+2}}{2(2\beta+1)} + \frac{\alpha r\mu^{\beta+1}t}{(\beta+1)(\beta+2)} + \alpha t^{\beta+2}r - \frac{r\alpha\mu t^{\beta+1}}{2} + \frac{r\alpha^2t^{2\beta+2}}{(\beta+1)} - \frac{r\alpha^2t^{\beta+1}\mu^{\beta+1}}{(\beta+1)(\beta+2)}$$

$$\int_{t_1}^{t_2} Q(t)e^{-rt}dt = D_0\mu\int_{t_1}^{t_2}[(t_2-t) + \frac{\alpha}{(\beta+1)}\left({t_2}^{\beta+1} - t^{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)}\ ({t_2}^{2\beta+1} - t^{2\beta+1})]e^{-\alpha t^{\beta}}\,e^{-rt}dt$$

$$= D_0\mu\int_{t_1}^{t_2}[(t_2-t) + \frac{\alpha}{(\beta+1)}\left({t_2}^{\beta+1} - t^{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)}\ \left({t_2}^{2\beta+1} - t^{2\beta+1}\right) - \alpha t^{\beta}(t_2-t) + \frac{\alpha^2}{(\beta+1)}t^{\beta}({t_2}^{\beta+1} - t^{\beta+1})]e^{-rt}\ dt$$

$$= D_0\ \mu\ \int_{t1}^{t_2}[(t_2-t) + \frac{\alpha}{(\beta+1)}\ \left({t_2}^{\beta+1} - t^{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)}\left(\,{t_2}^{2\beta+1} - t^{2\beta+1}\right) - \alpha\left(t_2t^{\beta} - t^{\beta+1}\right) + \frac{\alpha^2}{(\beta+1)}\ \left({t_2}^{\beta+1}t^{\beta}t^{2\beta+1}\right) - r\left(t_2t - \frac{t^2}{2}\right) - \frac{\alpha r}{(\beta+1)}\left({t_2}^{\beta+1}t - \frac{t^{\beta+2}}{2}\right) - \frac{\alpha^2 r}{2(2\beta+1)}\ \left({t_2}^{2\beta+1}t - t^{2\beta+2}\right) + \alpha r\left(t_2t^{\beta+1} - t^{\beta+2}\right) - \frac{\alpha^2 r}{(\beta+1)}\left({t_2}^{\beta+1}t^{\beta+1} - t^{2\beta+2}\right)]$$

$$= D_0\ \mu\ [t_2(t_2-t_1) - \frac{{t_2}^2}{2} + \frac{{t_1}^2}{2} + \frac{\alpha}{(\beta+1)}{t_2}^{\beta+1}(t_2-t_1) - \frac{\alpha}{(\beta+1)}\ \left(\frac{{t_2}^{\beta+2}-{t_1}^{\beta+2}}{(\beta+2)}\right) + \frac{\alpha^2}{2(2\beta+1)}{t_2}^{2\beta+1} + (t_2-t_1) - \frac{\alpha^2}{2(2\beta+1)} - \left(\frac{{t_2}^{2\beta+2}-t1^{2\beta+2}}{2(\beta+2)}\right) - \alpha t_2\left(\frac{{t_2}^{\beta+1}-{t_1}^{\beta+1}}{(\beta+1)}\right) + \alpha\left(\frac{{t_2}^{\beta+2}-{t_1}^{\beta+2}}{(\beta+2)}\right) + \frac{\alpha^2}{(\beta+1)}{t_2}^{\beta+1}\left(\frac{{t_2}^{\beta+2}-{t_1}^{\beta+1}}{(\beta+1)}\right) - \frac{\alpha^2}{(\beta+1)}\left(\frac{{t_2}^{2\beta+2}-{t_1}^{2\beta+2}}{(2\beta+2)}\right) - rt_2(t_2-t_1) + r\left(\frac{{t_2}^2-{t_1}^2}{2}\right) - \frac{r\alpha}{(\beta+1)}{t_2}^{\beta+1}(t_2-t_1) + \frac{r\alpha}{(\beta+1)}\ \left(\frac{{t_2}^{\beta+3}-{t_1}^{\beta+3}}{(\beta+3)}\right) - \frac{\alpha^2 r}{2(2\beta+1)}{t_2}^{2\beta+1}((*$$

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$$t_2-t_1) + \frac{\alpha^2 r}{2(2\beta+1)}(\frac{{t_2}^{2\beta+3}-{t_1}^{2\beta+3}}{(2\beta+3)}) + \alpha rt_2(\frac{{t_2}^{\beta+2}-{t_1}^{\beta+2}}{(\beta+2)} - \alpha r\left(\frac{{t_2}^{\beta+3}-{t_1}^{\beta+3}}{(\beta+3)}\right) - \frac{\alpha^2 r}{(\beta+1)}{t_2}^{\beta+1}\left(\frac{{t_2}^{\beta+2}-{t_1}^{\beta+2}}{(\beta+2)}\right) + \frac{\alpha^2 r}{(\beta+1)}(\frac{{t_2}^{2\beta+2}-{t_1}^{2\beta+2}}{(2\beta+3)})]$$

$$=D_0\ \mu[\frac{t_2^2}{2} - t_2t_1 + \frac{t_1^2}{2} + \frac{(\alpha)t_2^{\beta+2}\beta}{(\beta+1)(\beta+2)} - \frac{\alpha t_1t_2^{\beta+1}}{(\beta+1)} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2t_2^{2\beta+1}t_1}{2(2\beta+1)} + \frac{\alpha^2t_2^{2\beta+2}(\beta+3)}{2(2\beta+2)(\beta+1)} + \frac{\alpha^2t_1^{2\beta+2}(5\beta+3)}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha t_2t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha^2t_2^{\beta+1}t_1^{\beta+1}}{(\beta+1)^2} - \frac{rt_2^2}{2} + rt_2t_1 - \frac{rt_1^2}{2} - \frac{r\alpha t_2^{\beta+2}}{(\beta+1)} + \frac{r\alpha t_2^{\beta+1}t_1}{(\beta+1)} + \frac{r\alpha t_1^{\beta+3}\beta}{(\beta+3)(\beta+1)} - \frac{\alpha^2t_2^{2\beta+2}r}{2(2\beta+1)} + \frac{r\alpha t_2^{\beta+3}(2\beta+3)}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha^2\beta t_2^{2\beta+3}r}{2(2\beta+1)\,(\beta+2)(2\beta+3)} - \frac{\alpha^2(5\beta+3)\,t_1^{2\beta+3}r}{2(2\beta+1)\,(\beta+1)(2\beta+3)} - \frac{\alpha t_2t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2t_2^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)}]$$

Therefore, the total inventory in [0, t_2] is given by

$$\int_0^{t_2} Q(t)e^{-rt}dt = (\gamma-1)\,D_0[\frac{\mu^3}{6} - \frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(\beta+2)(2\beta+2)(2\beta+3)} - \frac{r\mu^2}{8} + \frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{2(\beta+2)(2\beta+2)(2\beta+4)}] + D_0\ \mu\ (\gamma-1)\left[\frac{{t_1}^2}{2} - \frac{\mu t_1}{2} - \frac{\alpha\beta {t_1}^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\beta\mu^{\beta+2}}{2(\beta+1)(\beta+2)} - \frac{\alpha^2(3\beta+1){t_1}^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha^2\beta\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+2)} - \frac{\alpha\mu^{\beta+1}t_1}{(\beta+1)(\beta+2)} - \frac{\alpha^2\mu^{2\beta+1}t_1}{2(2\beta+1)(2\beta+1)} + \frac{\alpha^2{t_1}^{\beta+1}\mu^{\beta+1}}{(\beta+1)^2(\beta+2)} - \frac{r{t_1}^3}{3} + \frac{r\mu^3}{12} + \frac{r{t_1}^2\mu}{4} + \frac{\alpha r{t_1}^{\beta+3}\beta}{(\beta+1)(\beta+3)} + \frac{r\alpha\beta\mu^{\beta+3}}{2(\beta+1)(\beta+3)(\beta+2)} - \frac{r\alpha^2{t_1}^{2\beta+3}}{2(2\beta+1)(2\beta+3)} + \frac{r\alpha\mu^{\beta+1}{t_1}^2}{2(\beta+1)(\beta+2)} + \frac{r\alpha^2\mu^{2\beta+1}{t_1}^2}{4(2\beta+1)(2\beta+2)} + \frac{r\alpha {t_1}^{(\beta+3)}}{(\beta+3)} -\right.$$

$$\frac{\alpha\mu r {t_1}^{\beta+2}}{2(\beta+2)} + \frac{\alpha^2 r {t_1}^{2\beta+3}}{(\beta+1)(2\beta+3)} - \frac{r\alpha^2 {t_1}^{\beta+2}\mu^{\beta+1}}{(\beta+1)(\beta+2)^2} + \frac{r\alpha^2\mu^{2\beta+3}(-7\beta^2-12\beta-4)}{8(\beta+1)(\beta+2)^2(2\beta+3)} + \frac{\alpha\mu t_1^{\beta+1}}{2(\beta+1)}\Big] + D_0\,\mu[\frac{t_2^2}{2} - t_2 t_1 + \frac{t_1^2}{2} + \frac{(\alpha)t_2^{\beta+2}\beta}{(\beta+1)(\beta+2)} - \frac{\alpha t_1 t_2^{\beta+1}}{(\beta+1)} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2 t_2^{2\beta+1}t_1}{2(2\beta+1)} + \frac{\alpha^2 t_2^{2\beta+2}(\beta+3)}{2(2\beta+2)(\beta+1)} + \frac{\alpha^2 t_1^{2\beta+2}(5\beta+3)}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha t_2 t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha^2 t_2^{\beta+1}t_1^{\beta+1}}{(\beta+1)^2} - \frac{rt_2^2}{2} + rt_2t_1 - \frac{rt_1^2}{2} - \frac{r\alpha t_2^{\beta+2}}{(\beta+1)} + \frac{r\alpha t_2^{\beta+1}t_1}{(\beta+1)} + \frac{r\alpha t_1^{\beta+3}\beta}{(\beta+3)(\beta+1)} - \frac{\alpha^2 t_2^{2\beta+2}r}{2(2\beta+1)} + \frac{r\alpha t_2^{\beta+3}(2\beta+3)}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha^2\beta t_2^{2\beta+3}r}{2(2\beta+1)(\beta+2)(2\beta+3)} - \frac{\alpha^2(5\beta+3)\,t_1^{2\beta+3}r}{2(2\beta+1)(\beta+1)(2\beta+3)} - \frac{\alpha t_2 t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t_2^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)}] \quad (19)$$

Total number of deteriorated items over the period $[0,t_2]$ is given by

Production in $[0,\mu]$+ Production in $[\mu,t_1]$ – Demand in $[0,\mu]$ –Demand in $[\mu, t_2]$

$$=\gamma \int_o^{\mu} Dote^{-rt}dt + \gamma\int_o^{t_1} D_0\,\mu e^{-rt}dt - D_0\int_o^{\mu} te^{-rt}dt - \int_{\mu}^{t_2} Do\mu e^{-rt}dt$$

$$=\gamma D_0\,[\frac{\mu^2}{2} - \frac{r\mu^3}{3}] + \gamma D_0\mu\left(t_1 - \mu - r\frac{t_1^2}{2} + r\frac{\mu^2}{2}\right) - D_0\left[\frac{\mu^2}{2} - \frac{r\mu^3}{3}\right] - D_0\mu[(t_2-\mu) - r\frac{t_2^2}{2} + r\frac{\mu^2}{2}]$$

$$=\frac{1}{2}\gamma D_0\mu\left[2t_1 - \mu - rt_1^2 + \frac{r\mu^2}{3}\right] - \frac{1}{2}D_0\,\mu\left[2t_2 - \mu - rt_2^2 + \frac{r\mu^2}{3}\right] \quad (20)$$

The cost of production in $[u, u+du]$ *is*

$$\text{Kv } du = \frac{\alpha_1\gamma}{R^{s-1}} \quad (21)$$

Hence the production cost over the period $[o, t_1]$ is given by

$$\int_o^{t_1}\text{Kv}e^{-ru}du = \int_0^{\mu}\text{Kv}e^{-ru}du + \int_{\mu}^{t_1}\text{Kv}e^{-ru}du$$

$$= \int_o^{\mu}\frac{\alpha_1\gamma}{R^{s-1}}e^{-ru}du + \int_{\mu}^{t_1}\frac{\alpha_1\gamma}{R^{s-1}}e^{-ru}du$$

$$= \alpha_1\gamma {D_0}^{1-s}\,[\int_o^{\mu}[u^{1-s}(1-ru) + \int_{\mu}^{t_1}u^{1-s}(1-ru)du]$$

$$=\alpha_1\gamma {D_0}^{1-s}[\frac{\mu^{2-s}}{(2-s)} - \frac{\mu^{1-s}}{(1-s)}] + \mu^{1-s}[(t_1-\mu) - \left(\frac{t_1^2}{2} - \frac{\mu^2}{2}\right)]$$

$$=\alpha_1\gamma {D_0}^{1-s}\left[\frac{(s-1)\mu^{2-s}+(2-s)\mu^{1-s}t_1}{(2-s)}\right] + \alpha_1\gamma r{D_0}^{1-s}[\frac{\mu^{3-s}}{2} - \frac{{t_1}^2}{2}\,\mu^{1-s} - \frac{\mu^{1-s}}{(1-s)}] \quad (22)$$

The total average inventory cost X is given by

X= Inventory Cost + Deterioration Cost+ Production Cost+ Shortage Cost + Lost Sale Cost

$$X = \frac{1}{t_2}[\,c_1\,\{(\gamma\text{-}1)\,D_0\,(\frac{\mu^3}{6} - \frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(\beta+2)(2\beta+2)(2\beta+3)} - \frac{r\mu^2}{8} + \frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{2(\beta+2)(2\beta+2)(2\beta+4)}) + D_0\,\mu(\gamma\text{-}1)\,(\frac{{t_1}^2}{2} - \frac{\mu t_1}{2} - \frac{\alpha\beta {t_1}^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\beta\mu^{\beta+2}}{2(\beta+1)(\beta+2)} - \frac{\alpha^2(3\beta+1){t_1}^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha^2\beta\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+2)} - \frac{\alpha\mu^{\beta+1}t_1}{(\beta+1)(\beta+2)} - \frac{\alpha^2\mu^{2\beta+1}t_1}{2(2\beta+1)(2\beta+2)} + \frac{\alpha^2{t_1}^{\beta+1}\mu^{\beta+1}}{(\beta+1)^2(\beta+2)} - \frac{r{t_1}^3}{3} + \frac{r\mu^3}{12} + \frac{r{t_1}^2\mu}{4} + \frac{\alpha r{t_1}^{\beta+3}\beta}{(\beta+1)(\beta+3)} + \frac{r\alpha\beta\mu^{\beta+3}}{2(\beta+1)(\beta+3)(\beta+2)} - \frac{r\alpha^2{t_1}^{2\beta+3}}{2(2\beta+1)(2\beta+3)} + \frac{r\alpha\mu^{\beta+1}{t_1}^2}{2(\beta+1)(\beta+2)} + \frac{r\alpha^2\mu^{2\beta+1}{t_1}^2}{4(2\beta+1)(2\beta+2)} + \frac{r\alpha{t_1}^{(\beta+3)}}{(\beta+3)} - \frac{\alpha\mu r{t_1}^{\beta+2}}{2(\beta+2)} + \frac{\alpha^2 r{t_1}^{2\beta+3}}{(\beta+1)(2\beta+3)} - \frac{r\alpha^2{t_1}^{\beta+2}\mu^{\beta+1}}{(\beta+1)(\beta+2)^2} + \frac{r\alpha^2\mu^{2\beta+3}(-7\beta^2-12\beta-4)}{8(\beta+1)(\beta+2)^2(2\beta+3)} + \frac{\alpha\mu t_1^{\beta+1}}{2(\beta+1)}) + D_0\,\mu(\frac{t_2^2}{2} - t_2t_1 + \frac{t_1^2}{2} + \frac{(\alpha)t_2^{\beta+2}\beta}{(\beta+1)(\beta+2)} - \frac{\alpha t_1 t_2^{\beta+1}}{(\beta+1)} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2 t_2^{2\beta+1}t_1}{2(2\beta+1)} + \frac{\alpha^2 t_2^{2\beta+2}(\beta+3)}{2(2\beta+2)(\beta+1)} + \frac{\alpha^2 t_1^{2\beta+2}(5\beta+3)}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha t_2 t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha^2 t_2^{\beta+1}t_1^{\beta+1}}{(\beta+1)^2} - \frac{rt_2^2}{2} + rt_2t_1 - \frac{rt_1^2}{2} - \frac{r\alpha t_2^{\beta+2}}{(\beta+1)} + \frac{r\alpha t_2^{\beta+1}t_1}{(\beta+1)} + \frac{r\alpha t_1^{\beta+3}\beta}{(\beta+3)(\beta+1)} - \frac{\alpha^2 t_2^{2\beta+2}r}{2(2\beta+1)} - \frac{r\alpha t_2^{\beta+3}(2\beta+3)}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha^2\beta t_2^{2\beta+3}r}{2(2\beta+1)(\beta+2)(2\beta+3)} - \frac{\alpha^2(5\beta+3)\,t_1^{2\beta+3}r}{2(2\beta+1)(\beta+1)(2\beta+3)} - \frac{\alpha t_2 t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t_2^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)})\} + c_3\{\frac{1}{2}\gamma D_0\mu\left(2t_1 - \mu - rt_1^2 + \frac{r\mu^2}{3}\right) - \frac{1}{2}D_0\,\mu(2t_2 - \mu - rt_2^2 + \frac{r\mu^2}{3})\} - c_4 D_0\,\mu\left\{Tt_2 - \frac{t_2^2}{2} - \frac{T^2}{2} - \delta\left(\frac{3t_2T^2}{2} - \frac{3t_2^2T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2}\right) - r\frac{T^2t_2}{2} + r\frac{t_2^3}{2} + r\frac{T^3}{3} + \delta r\,(T-t_2)\left(\frac{t_2T^2}{2} - \frac{t_2^3}{6} - \frac{T^3}{3}\right)\right\} + c_5 D_0\,\mu\delta\left(\frac{3T^2t_2}{2} - \frac{3t_2^2T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2} - \frac{5rt_2T^3}{6} - \frac{rt_2^4}{6} + \frac{rTt_2^3}{6} + \frac{rT^4}{3} + \frac{rt_2^2T^2}{2}\right) + \alpha_1\gamma{D_0}^{1-s}\{\frac{(s-1)\mu^{2-s}+(2-s)\mu^{1-s}t_1}{(2-s)}\} + \alpha_1\gamma r{D_0}^{1-s}\{\frac{\mu^{3-s}}{2} - \frac{{t_1}^2}{2}\,\mu^{1-s} - \frac{\mu^{1-s}}{(1-s)}\}] \quad (23)$$

Optimum values of t_1 and t_2 for minimum average cost x are the solutions of the equations

$$\frac{\partial X}{\partial t_1} = 0 \text{ and } \frac{\partial X}{\partial t_2} = 0$$

Provided they satisfy the sufficient conditions

$$\frac{\partial^2X}{\partial t_1^2} > 0, \frac{\partial^2X}{\partial t_2^2} > 0 \text{ and } \frac{\partial^2X}{\partial t_1^2}\frac{\partial^2X}{\partial t_2^2} - \left(\frac{\partial^2X}{\partial t_1\partial t_2}\right)^2 > 0$$

$$\frac{\partial x}{\partial t_1} = 0 \text{ and } \frac{\partial x}{\partial t_2} = 0 \text{ gives}$$

$$C_1\left\{D_0\mu(\gamma-1)\left(t_1-\frac{\alpha\beta t_1^{\beta+1}}{(\beta+1)}-\frac{\alpha^2(3\beta+1)t_1^{2\beta+1}}{2(2\beta+1)(\beta+1)}-\frac{\alpha\mu^{\beta+1}}{(\beta+1)(\beta+2)}-\frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)}-\frac{\mu}{2}+\frac{\alpha^2 t_1^{\beta}\mu^{\beta+1}}{(\beta+1)(\beta+2)}-rt_1^2+\frac{rt_1\mu}{2}+\frac{\alpha r t_1^{\beta+2}\beta}{(\beta+1)}-\frac{r\alpha^2 t_1^{2\beta+2}}{2(2\beta+1)}+\frac{r\alpha\mu^{\beta+1}t_1}{(\beta+1)(\beta+2)}+\frac{r\alpha^2\mu^{2\beta+1}t_1}{2(2\beta+1)(2\beta+2)}+r\alpha t_1^{\beta+2}-\frac{\alpha\mu r t_1^{\beta+1}}{2}+\frac{\alpha^2 r t_1^{2\beta+2}}{(\beta+1)}-\frac{r\alpha^2 t_1^{\beta+1}\mu^{\beta+1}}{(\beta+1)(\beta+2)}+\frac{\alpha\mu t_1^{\beta}}{2}\right)+D_0\mu\left(-t_2+t_1-\frac{\alpha t_2^{\beta+1}}{(\beta+1)}-\frac{\alpha\beta t_1^{\beta+1}}{(\beta+1)}-\frac{\alpha^2 t_2^{2\beta+1}}{2(2\beta+1)}+\frac{\alpha^2 t_1^{2\beta+1}(5\beta+3)}{2(2\beta+1)(\beta+1)}+\alpha t_2 t_1^{\beta}-\frac{\alpha^2 t_2^{\beta+1}t_1^{\beta}}{(\beta+1)}+rt_2-rt_1+\frac{r\alpha t_2^{\beta+1}}{(\beta+1)}+\frac{r\alpha t_1^{\beta+2}\beta}{(\beta+1)}-\frac{\alpha^2(5\beta+3)t_1^{2\beta+2}r}{2(2\beta+1)(\beta+1)}-\alpha t_2 t_1^{\beta+1}+\frac{\alpha^2 t_2^{\beta+1}t_1^{\beta+1}}{(\beta+1)}\right)\right\}+c_3\gamma D_0\mu(1-rt_1)+\alpha_1\gamma D_0^{1-s}\mu^{1-s}-\alpha_1\gamma r D_0^{1-s}t_1\mu^{1-s}=0 \quad (24)$$

and

$$C_1 D_o\mu\left(t_2-t_1+\frac{\alpha\beta t_2^{\beta+1}}{(\beta+1)}-\alpha t_1 t_2^{\beta}-\frac{\alpha^2 t_1 t_2^{2\beta}}{2}+\frac{\alpha^2(\beta+3)t_2^{2\beta+1}}{2(\beta+1)}+\frac{\alpha t_1^{\beta+1}}{(\beta+1)}-\frac{\alpha^2 t_1^{\beta+1}t_2^{\beta}}{(\beta+1)}-rt_2+rt_1-\frac{r\alpha(\beta+2)t_2^{\beta+1}}{(\beta+1)}+r\alpha t_1 t_2^{\beta}-\frac{r\alpha^2(2\beta+2)t_2^{2\beta+1}}{2(2\beta+1)}+\frac{r\alpha(2\beta+3)t_2^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{3\alpha^2\beta r t_2^{2\beta+2}}{2(2\beta+1)(\beta+2)}-\frac{\alpha t_1^{\beta+2}}{(\beta+2)}+\frac{\alpha^2 t_2^{\beta}t_1^{\beta+2}}{(\beta+2)}\right)-C_3 D_o\mu(1-rt_2)-C_4 D_o\mu\{(T-t_2)-\delta(\tfrac{3}{2}T^2-3t_2T+\frac{3t_2^2}{2})-\frac{rT^2}{2}+\frac{3rt_2^2}{2}-\delta r t_2 T^2+\tfrac{2}{3}\delta r t_2^3+\tfrac{5}{6}\delta r T^3-\frac{\delta r T t_2^2}{2}\}+C_5 D_o\mu\delta\left(\tfrac{3}{2}T^2-3t_2T+\tfrac{3}{2}t_2^2-\frac{5rT^3}{6}-\tfrac{2}{3}rt_2^3+\tfrac{1}{2}rTt_2^2+rt_2T^2\right)-X=0 \quad (25)$$

Case–II ($t_1 \le \mu \le t_2$)

The production starts with zero stock level at t=0. Production begins at t=0 and continues up to $t=t_1$and stops as soon as the stock level becomes L at t= t_2. Because of reasons of market demand and deterioration of items, the inventory level decreases till it becomes again zero at t= t_2 .After time t= t_2, another important factor occurs which is shortages. After that period, the cycle repeats itself.

Let Q(t) be the inventory level of the system at any time t ($0 \le t \le t_2$). The differential equations governing the system in the interval [0, t_2] are given by

$$\frac{dQ(t)}{dt}+\alpha\beta\, t^{\beta-1}\, Q(t)=K-F(t) \qquad 0\le t\le t_1 \qquad (26)$$

with the condition Q (0) = 0, Q (t_1) = L

$$\frac{dQ(t)}{dt}+\alpha\beta\, t^{\beta-1}\, Q(t)=-F(t) \qquad t_1\le t\le \mu \qquad (27)$$

with the condition Q (t_1) = L

$$\frac{dQ(t)}{dt}+\alpha\beta\, t^{\beta-1}\, Q(t)=-F(t) \qquad \mu\le t\le t_2 \qquad (28)$$

with the condition Q (t_2) = 0

$$\frac{dQ}{dt}=-e^{-\delta(T-t)}\,F(t) \qquad t_2\le t\le T \qquad (29)$$

with the condition Q (t_2) = 0

using ramp type function F (t) equations (26),(27)(28),(29) become respectively

$$\frac{dQ(t)}{dt}+\alpha\beta\, t^{\beta-1}\, Q(t)=(\gamma-1)D_0 t \qquad 0\le t\le t_1 \qquad (30)$$

with the condition Q (0) = 0, Q (t_1) = L

$$\frac{dQ(t)}{dt}+\alpha\beta\, t^{\beta-1}\, Q(t)=-D_0 t \qquad t_1\le t\le \mu \qquad (31)$$

with the condition Q (t_1) = L

$$\frac{dQ(t)}{dt}+\alpha\beta\, t^{\beta-1}\, Q(t)=-D_0\,\mu \qquad \mu\le t\le t_2 \qquad (32)$$

with the condition Q (t_2) = 0

$$\frac{dQ}{dt}=-e^{-\delta(T-t)}\,D_0\mu \qquad t_2\le t\le T \qquad (33)$$

with the condition Q (t_2) = 0

The solution of equation (30) is given by the expression (11) and we have

$$e^{\alpha t^{\beta}}\ Q(t) = (\gamma-1)\ D_0 \left(\frac{t^2}{2} + \frac{\alpha\, t^{\beta+2}}{\beta+2} + \alpha^2\ \frac{t^{2\beta+2}}{2(2\beta+2)} + \ \right) + C$$

With the condition Q(0) = 0, we get

$$Q(t) = (\gamma-1)\ D_0\, e^{-\alpha t^{\beta}} \left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)} + \ \right) \qquad 0 \le t \le t_1 \qquad (34)$$

Using boundary condition $Q(t_1) = L$ in (34) we get

$$L = (\gamma-1)\ D_0\, e^{-\alpha t_1^{\beta}} \left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} + \ \right) \qquad (35)$$

Therefore the solution of equation (31) is given by

$$\frac{d\,Q(t)}{dt} + \alpha\beta\ t^{\beta-1}\ Q(t) = -\,D_0 t$$

$$e^{\alpha t^{\beta}}\ Q(t) \quad = -\mathrm{D}_0 \int \mathrm{te}^{\alpha+\beta}\ \mathrm{dt} + \mathrm{C}$$

$$= -D_0 \int t\left(1+\alpha\, t^{\beta} + \frac{\alpha^2\, t^{2\beta}}{2} + \ \right) dt + C$$

$$= -D_0 \left(\frac{t^2}{2} + \frac{\alpha\ t^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)} + \ \right) + C$$

Using condition $Q(t_1) = L$

$$Le^{\alpha t_1 \beta} \quad = -D_0 \left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} + \ \right) + C$$

$$C = Le^{\alpha t_1^{\beta}}\ + D_0 \left(\frac{t_1^2}{2} + \frac{\alpha\, t_1^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} + \ \right)$$

$$e^{\alpha t^{\beta}}\ Q(t) = -D_0 \left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)} + \ \right) + (\gamma-1)\ D_0 e \left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} + \ \right)$$

$$e^{\alpha t^{\beta}}\ Q(t) = -D_0 \left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)} + \ \right) + Le^{\alpha t_1^{\beta}} + D_0 \left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2t)} \right)$$

$$+ D_0 \left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} + \ \right)$$

$$Q(t) = -D_0 e^{-\alpha t^{\beta}} \left(\frac{t^2}{2} + \frac{2t^{\beta+2}}{(\beta+2)} + \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)} + \ \right) + \gamma\, D_0 e^{-\alpha t^{\beta}} \left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} + \ \right),\ t_1 \le t \le \mu \qquad (36)$$

Using boundary condition $Q(t_2) = 0$, the solution of equation (32) is given by

$$\frac{\mathrm{d\,Q(t)}}{\mathrm{dt}} + \alpha\beta\, \mathrm{t}^{\beta-1}\ \mathrm{Q(t)} = -\,\mathrm{D}_0\mu \qquad \mu \le t \le t_2$$

$$Q(t)\, e^{\alpha t^{\beta}} \quad = -\,D_0\mu \int e^{\alpha+\beta}\ dt + C$$

$$= -D_0\mu \left(t + \frac{\alpha\ t^{\beta+1}}{(\beta+1)} + \frac{\alpha^2\ t^{2\beta+1}}{2(2\beta+1)} + \ \right) + C$$

$$\mathrm{C} = D_0\mu \left(t_2 + \frac{\alpha t_2^{\beta+1}}{(\beta+1)} + \frac{\alpha^2\ t_2^{2\beta+1}}{2(2\beta+1)} + \ \right)$$

$$Q(t)\, e^{\alpha t^{\beta}} = -D_0\mu \left(t + \frac{\alpha t^{\beta+1}}{(\beta+1)} + \frac{\alpha^2\, t^{2\beta+1}}{2(2\beta+1)} + \ \right) + D_0\mu \left(t_2 + \frac{\alpha t_2^{\beta+1}}{(\beta+1)} + \frac{\alpha^2\, t_2^{2\beta+1}}{2(2\beta+1)} + \ \right)$$

$$\mathrm{Q(t)} = D_0\mu e^{-\alpha t^{\beta}} \left((t_2 - t) + \frac{\alpha}{(\beta+1)} (t_2^{\beta+1} - t^{\beta+1}) + \frac{\alpha^2}{2(2\beta+1)} (t_2^{2\beta+1} - t^{2\beta+2}) + \ \right),\ \ \mu \le t \le t_2 \qquad (37)$$

The solution of equation (33) is given by

$$\frac{dQ(t)}{dt} = -D_0\, \mu\, e^{-\delta(T-t_2)} \qquad t_2 \le t \le T$$

with boundary condition $Q(t_2) = 0$

$$Q\,(t) = -D_0\, \mu \int [1-\delta(T-t_2)]dt + c$$

$$Q\,(t) = -D_0\, \mu\, [t-\delta(T-t_2)t] + c$$

By using $Q(t_2)$ =0, we get

$$Q(t) = D_0\, \mu[(t_2-t)-\delta(T-t_2)(t_2-t)] \tag{38}$$

$$\frac{dQ(t)}{dt} = -e^{-\delta}\,(T-t_2)\,D_0\mu$$

Total inventory over the period $[0, t_2]$ is

$$\int_0^{t_2} Q(t)\,e^{-rt}\,dt = \int_0^{t_1} Q(t)\,e^{-rt}\,dt + \int_{t_1}^{\mu} Q(t)\,e^{-rt}\,dt + \int_{\mu}^{t_2} Q(t)\,e^{-rt}\,dt$$

$$\int_0^{t_1} Q(t)\,e^{-rt}\,dt = (\gamma-1)\,D_0 \int_0^{t_1} e^{-\alpha t^{\beta}} \left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + \right) e^{-rt}\,dt$$

$$\int_0^{t_1} Q(t)\,e^{-rt}\,dt = (\gamma-1)\,D_0 \int_0^{t_1} (1-\alpha t^{\beta}) \left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + \right) e^{-rt}\,dt$$

$$=(\gamma-1)\,D_0 \int_0^{t_1}\left(\frac{t^2}{2} - \frac{\alpha\beta\, t^{\beta+2}}{2(\beta+2)} - \frac{\alpha^2(3\beta+2)\,t^{2\beta+2}}{2(2\beta+2)(\beta+2)} - \frac{rt^3}{2} + \frac{\alpha\beta r t^{\beta+3}}{2(\beta+2)} + \frac{r\alpha^2\,(3\beta+2)\,t^{2\beta+3}}{2\,(2\beta+2)\,(\beta+2)}\right)$$

$$=(\gamma-1)D_0\left[\frac{t_1^3}{6} - \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)\,(\beta+3)} - \frac{\alpha^2\,(3\beta+2)\,t_1^{2\beta+3}}{2(2\beta+2)\,(\beta+2)\,(2\beta+3)} - \frac{rt_1^4}{8} + \frac{\alpha\beta r t_1^{\beta+4}}{2(\beta+2)\,(\beta+4)} + \frac{r\alpha^2\,(3\beta+2)\,t_1^{2\beta+4}}{2(2\beta+2)\,(\beta+2)\,(2\beta+4)}\right]$$

$$\int_{t_1}^{\mu} Q(t)\,e^{-rt}\,dt = D_0 \int_{t_1}^{\mu} e^{-\alpha t^{\beta}} \quad \left(\frac{\gamma t_1^2}{2} + \frac{\alpha\gamma\, t_1^{\beta+2}}{(\beta+2)} + \frac{\gamma\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} - \frac{t^2}{2} - \frac{\alpha t^{\beta+2}}{(\beta+2)} - \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)}\right)(1-rt)$$

$$= D_0 \int_{t_1}^{\mu} [\frac{\gamma t_1^2}{2} + \frac{\alpha\gamma\, t_1^{\beta+2}}{(\beta+2)} + \frac{\gamma\alpha^2\, t_1^{2\beta+2}}{2(2\beta+2)} - \frac{t^2}{2} - \frac{\alpha t^{\beta+2}}{(\beta+2)} - \frac{\alpha^2\, t^{2\beta+2}}{2(2\beta+2)} - \frac{\alpha\,\gamma\, t^{\beta}\, t_1^2}{2} - \frac{\alpha^2\gamma\, t^{\beta}\ t_1^{\beta+1}}{(\beta+2)} +$$

$$\frac{\alpha\, t^{\beta+2}}{2} + \frac{\alpha^2\, t^{2\beta+2}}{(\beta+2)} - \frac{r\,\gamma t_1^2 t}{2} - \frac{r\,\alpha\gamma\, t_1^{\beta+2}}{(\beta+2)} - \frac{r\,\gamma\alpha^2\, t_1^{2\beta+2} t}{2(2\beta+2)} + \frac{r\,t^3}{2} + \frac{r\,\alpha t^{\beta+3}}{(\beta+2)} + \frac{r\,\alpha^2\, t^{2\beta+3}}{2(2\beta+2)} +$$

$$\frac{r\,\alpha\,\gamma\, t^{\beta+1}\, t_1^2}{2} + \frac{r\,\alpha^2\,\gamma\, t^{\beta+1}\ t_1^{\beta+2}}{(\beta+2)} - \frac{r\,\alpha\, t^{\beta+3}}{2} - \frac{r\,\alpha^2\, t^{2\beta+3}}{(\beta+2)}]$$

$$=D_0\left[\frac{\gamma\,\mu\, t_1^2}{2} - \frac{\gamma t_1^3}{2} + \frac{\alpha\gamma\, t_1^{\beta+2}\,\mu}{(\beta+2)} + \frac{\alpha\beta\gamma\, t_1^{\beta+3}}{2(\beta+2)\,(\beta+1)} + \frac{\alpha^2\gamma\, t_1^{2\beta+2}\,\mu}{2(2\beta+2)} - \frac{\alpha\beta\, t_1^{\beta+3}}{2(\beta+2)\,(\beta+3)} - \frac{\mu^3}{6} + \frac{t_1^3}{6} +\right.$$

$$\frac{\alpha\beta\,\mu^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha^2\,(3\beta+2)\,\mu^{2\beta+3}}{2\,(2\beta+2)(2\beta+3)(\beta+2)} - \frac{\alpha\gamma\,\mu^{\beta+1}\ t_1^2}{2(\beta+1)} - \frac{\alpha^2\gamma\,\mu^{\beta+1}\, t_1^{\beta+2}}{(\beta+1)\,(\beta+2)}$$

$$-\frac{\alpha^2\,(3\beta+2)\ t_1^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)} - \frac{\alpha^2\gamma\,(\beta-2)\,t_1^{2\beta+3}}{4(\beta+1)\,(\beta+2)} - \frac{r\gamma\, t_1^2\ \mu^2}{4} + \frac{r\gamma\, t_1^4}{4} - \frac{r\alpha\gamma\, t_1^{\beta+2}\ \mu^2}{2\,(\beta+2)} + \frac{r\alpha\gamma \quad t_1^{\beta+4}}{2(\beta+2)}$$

$$-\frac{r\gamma\alpha^2\, t_1^{2\beta+2}\ \mu^2}{4(2\beta+2)} + \frac{r\gamma\,\alpha^2\, t_1^{2\beta+4}}{4(2\beta+2)} + \frac{r\mu^4}{8} - \frac{rt_1^4}{8} + \frac{r\alpha\,\mu^{\beta+4}}{(\beta+2)\,(\beta+4)} - \frac{r\alpha\, t_1^{\beta+4}}{(\beta+2)\,(\beta+4)} + \frac{r\alpha^2\ \mu^{2\beta+4}}{8\,(\beta+1)\,(\beta+2)}$$

$$-\frac{r\alpha^2\, t_1^{2\beta+4}}{8\,(\beta+1)\,(\beta+2)} + \frac{r\alpha\gamma\,\mu^{\beta+2}\, t_1^2}{2(\beta+2)} - \frac{r\alpha\gamma \quad t_1^{\beta+4}}{2(\beta+2)} - \frac{r\gamma\alpha^2\, t_1^{2\beta+4}}{(\beta+2)^2} + \frac{r\gamma\alpha^2\, t_1^{\beta+2}\mu^{\beta+2}}{(\beta+2)^2} - \frac{r\alpha\,\mu^{\beta+4}}{2\,(\beta+4)} + \frac{r\alpha\, t_1^{\beta+4}}{2\,(\beta+4)} - \frac{r\alpha^2\ \mu^{2\beta+4}}{(2\beta+4)(\beta+2)} +$$

$$\left.\frac{r\alpha^2\, t_1^{2\beta+4}}{(2\beta+4)\,(\beta+2)}\right]$$

$=\mathrm{D}_0\ [\frac{\gamma\mu t_1^2}{2}-\frac{\gamma t_1^3}{2}+\frac{\alpha\mu\gamma t_1^{\beta+2}}{(\beta+2)}-\frac{\alpha\gamma\beta t_1^{\beta+3}}{2(\beta+2)(\beta+1)}+\frac{\alpha^2\gamma t_1^{2\beta+2}\mu}{2(2\beta+2)}-\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\mu^3}{6}+\frac{t_1^3}{6}+\frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)}-\frac{\alpha\gamma\mu^{\beta+1}t_1^2}{2(\beta+1)}-$
$\frac{\alpha^2\gamma\mu^{\beta+1}t_1^{\beta+2}}{(\beta+2)(\beta+2)}-\frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)}-\frac{\alpha^2\gamma(\beta-2)t_1^{2\beta+3}}{2(\beta+1)(\beta+2)}-\frac{r\gamma\mu^2t_1^2}{4}+\frac{r\gamma t_1^4}{4}-\frac{r\alpha\gamma t_1^{\beta+2}\mu^2}{2(\beta+2)}-\frac{r\gamma\alpha^2t_1^{2\beta+2}\mu^2}{8(\beta+1)}+\frac{r\gamma\alpha^2(\beta^2-4\beta-4)t_1^{2\beta+4}}{8(\beta+1)\ (\beta+2)^2}+\frac{r\mu^4}{8}-$
$\frac{rt_1^4}{8}-\frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)}+\frac{r\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)}-\frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{8(\beta+1)(\beta+2)^2}+\frac{r\alpha^2(3\beta+2)t_1^{2\beta+4}}{8(\beta+2)^2(\beta+1)}+\frac{r\alpha\gamma\mu^{\beta+2}t_1^2}{2(\beta+2)}+\frac{r\alpha^2\gamma t_1^{\beta+2}\ \mu^{\beta+2}}{(\beta+2)^2}]$

$$\int_\mu^{t_2} Q(t)\,e^{-rt}dt\ =D_0\mu\int_\mu^{t_2}e^{-\alpha t^\beta}\,[(t_2-t_1)\frac{\alpha}{(\beta+1)}\left(t_2^{\beta+1}-t^{\beta+1}\right)+\frac{\alpha^2}{2(2\beta+1)}\left(t_2^{2\beta+1}-t^{2\beta+1}\right)]\,e^{-rt}dt$$

$=D_0\mu\int_\mu^{t_2}\left(1-\alpha t^\beta\right)[(t_2-t_1)\frac{\alpha}{(\beta+1)}\left(t_2^{\beta+1}-t^{\beta+1}\right)+\frac{\alpha^2}{2(2\beta+1)}\left(t_2^{2\beta+1}-t^{2\beta+1}\right)]\ e^{-rt}dt$

$=D_0\mu\int_\mu^{t_2}[(t_2-t)+\frac{\alpha}{(\beta+1)}\left(t_2^{\beta+1}-t^{\beta+1}\right)+\frac{\alpha^2}{2(2\beta+1)}\left(t_2^{2\beta+1}-t^{2\beta+1}\right)-\alpha t^\beta t_2+\alpha t^{\beta+1}-\frac{\alpha^2}{(\beta+1)}t^\beta t_2^{\beta+1}+\frac{\alpha^2}{(\beta+1)}t^{2\beta+1}-$
$rt(t_2-t)-\frac{r\alpha}{(\beta+1)}\ (t_2^{\beta+1}-t^{\beta+2})\frac{r\alpha^2}{2(2\beta+1)}\left(t_2^{2\beta+1}t-t^{2\beta+2}\right)+\alpha\gamma t^{\beta+1}t_2-r\alpha t^{\beta+2}+\frac{\gamma\alpha^2}{(\beta+1)}t^{\beta+1}t_2^{\beta+1}-\frac{\gamma\alpha^2}{(\beta+1)}t^{2\beta+2}]$

$=\mathrm{D}_0\,\mu[t_2^2-t_2\,\mu-\frac{t_2^2}{2}+\frac{\mu^2}{2}+\frac{\alpha}{(\beta+1)}\left(t_2^{\beta+2}-t_2^{\beta+1}\mu-\frac{t_2^{\beta+2}}{\beta+2}+\frac{\mu^{\beta+2}}{(\beta+2)}\right)+\frac{\alpha^2}{2(2\beta+1)}\left(t_2^{2\beta+2}-t_2^{2\beta+1}\mu-\frac{t_2^{2\beta+2}}{(2\beta+2)}+\frac{\mu^{2\beta+2}}{(2\beta+2)}\right)-\frac{\alpha t_2^{\beta+2}}{(\beta+1)}+$
$\frac{\alpha\mu^{\beta+1}t_2}{(\beta+1)}+\frac{\alpha t_2^{\beta+2}}{(\beta+2)}-\frac{\alpha\mu^{\beta+2}}{(\beta+2)}-\frac{\alpha^2t_2^{2\beta+2}}{(\beta+1)^2}+\frac{\alpha^2\mu^{\beta+1}t_2^{\beta+1}}{(\beta+1)^2}+\frac{\alpha^2t_2^{2\beta+2}}{(\beta+1)(2\beta+2)}-\frac{\alpha^2\mu^{2\beta+1}}{(\beta+1)(2\beta+2)}-\frac{rt_2^3}{2}+\frac{r\mu^2t_2}{2}+\frac{rt_2^3}{3}-\frac{r\mu^3}{3}-\frac{r\alpha}{(\beta+1)}\Bigg(\frac{t_2^{\beta+3}}{2}-$
$t_2^{\beta+1}\frac{\mu^2}{2}-\frac{t_2^{\beta+3}}{\beta+3}+\frac{\mu^{\beta+3}}{(\beta+3)}\Bigg)-\frac{r\alpha^2}{2(2\beta+1)}\left(\frac{t_2^{2\beta+3}}{2}-t_2^{2\beta+1}\frac{\mu^2}{2}-\frac{t_2^{2\beta+3}}{2\beta+3}+\frac{\mu^{2\beta+3}}{2\beta+2}\right)+\frac{\alpha rt_2^{\beta+3}}{\beta+2}-\frac{\alpha r\mu^{\beta+2}t_2}{(\beta+2)}-\frac{\alpha rt_2^{\beta+3}}{\beta+3}+\frac{r\alpha\mu^{\beta+3}}{(\beta+3)}+\frac{r\alpha^2t_2^{2\beta+3}}{(\beta+1)(\beta+2)}-$
$\frac{r\alpha^2\mu^{\beta+2}t_2^{\beta+1}}{(\beta+1)(\beta+2)}-\frac{r\alpha^2t_2^{2\beta+3}}{(\beta+1)(2\beta+3)}+\frac{r\alpha^2\mu^{2\beta+3}}{(\beta+1)(2\beta+3)}]$

$=D_0\mu[(\frac{t_2^2}{2}-t_2\,\mu+\frac{\mu^2}{2})-\frac{\alpha\mu t_2^{\beta+1}}{(\beta+1)}+\frac{\alpha\beta t_2^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha^2(\beta-1)t_2^{2\beta+2}}{2(2\beta+2)(\beta+1)}-\frac{\alpha^2\mu t_2^{2\beta+1}}{2(2\beta+1)}-\frac{\alpha^2(3\beta+1)\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)}+\frac{\alpha^2\mu^{\beta+1}t_2^{\beta+1}}{(\beta+1)^2}+$
$\frac{\alpha\mu^{\beta+1}t_2}{(\beta+1)}-\frac{rt_2^3}{6}+\frac{r\mu^2t_2}{2}-\frac{r\mu^3}{3}-\frac{\alpha\beta rt_2^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{r\alpha\mu^2t_2^{\beta+1}}{2(\beta+1)}+\frac{\alpha r\beta\mu^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{r\alpha\mu^{\beta+2}t_2}{(\beta+2)}-\frac{r\alpha^2(\beta-2)t_2^{2\beta+3}}{4(\beta+2)(2\beta+3)}+\frac{r\alpha^2\mu^2t_2^{2\beta+1}}{4(2\beta+1)}+\frac{r\alpha^2(3\beta+1)\mu^{2\beta+3}}{2(\beta+1)(2\beta+3)(2\beta+1)}]$

Total inventory over the period $[0,\,t_2]$ *is* given by

$\int_\circ^{t_2}Q(t)e^{-rt}\,dt\ =(\gamma-1)\,D_0\left[\frac{t_1^3}{6}-\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(\beta+2)(2\beta+3)}-\frac{rt_1^4}{8}+\frac{\alpha\beta rt_1^{\beta+4}}{2(\beta+2)(\beta+4)}+\frac{r\alpha^2(3\beta+2)t_1^{2\beta+4}}{2(2\beta+2)(\beta+2)(2\beta+4)}\right]+D_0\left[\frac{\gamma\mu t_1^2}{2}-\right.$
$\frac{\gamma t_1^3}{2}+\frac{\alpha\gamma t_1^{\beta+2}\mu^2}{\beta+2}-\frac{\alpha\gamma\beta t_1^{\beta+3}}{2(\beta+2)(\beta+1)}+\frac{\alpha^2\gamma t_1^{2\beta+2}\mu}{2(2\beta+2)}-\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\mu^3}{6}+\frac{t_1^3}{6}+\frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)}-\frac{\alpha\gamma\mu^{\beta+1}t_1^2}{2(\beta+1)}-$
$\frac{\alpha^2\gamma\mu^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)}-\frac{\alpha^2\gamma(\beta-2)t_1^{2\beta+3}}{4(\beta+1)(\beta+2)}-\frac{r\gamma t_1^2\mu^2}{4}+\frac{r\gamma t_1^4}{4}-\frac{r\alpha\gamma t_1^{\beta+2}\mu^2}{2(\beta+2)}-\frac{r\gamma\alpha^2t_1^{2\beta+2}\mu^2}{8(\beta+1)}+\frac{r\gamma\alpha^2(\beta^2-4\beta-4)t_1^{2\beta+4}}{8(\beta+1)(\beta+2)^2}+\frac{r\mu^4}{8}-\frac{rt_1^4}{8}-$
$\left.\frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)}+\frac{r\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)}-\frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{8(\beta+1)(\beta+2)^2}+\frac{r\alpha^2(3\beta+2)\ t_1{}^{2\beta+4}}{8(\beta+2)^2(\beta+1)}+\frac{r\alpha\gamma\mu^{\beta+2}t_1^2}{2(\beta+2)}+\frac{r\alpha^2\gamma t_1^{\beta+2}\mu^{\beta+2}}{(\beta+2)^2}\right]+D_0\mu\left[\frac{t_2^2}{2}-t_2\,\mu+\frac{\mu^2}{2}\right]-\frac{\alpha\mu t_2^{\beta+1}}{(\beta+1)}+$
$\frac{\alpha\beta t_2^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\mu^{\beta+1}t_2}{(\beta+1)}+\frac{\alpha^2(\beta-1)t_2^{2\beta+2}}{2(2\beta+2)(\beta+1)}-\frac{\alpha^2\mu t_2{}^{2\beta+1}}{2(2\beta+1)}-\frac{\alpha^2(3\beta+1)\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)}+\frac{\alpha^2\mu^{\beta+1}t_2{}^{\beta+1}}{(\beta+1)^2}-\frac{rt_2^3}{6}+\frac{r\mu^2t_2}{2}-\frac{r\mu^3}{3}-$
$\frac{\alpha\beta rt_2^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{r\alpha\mu^2t_2^{\beta+1}}{2(\beta+1)}+\frac{\alpha r\beta\mu^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{r\alpha\mu^{\beta+2}t_2}{(\beta+2)}-\frac{r\alpha^2(\beta-2)t_2^{2\beta+3}}{4(\beta+2)(2\beta+3)}+\frac{r\alpha^2\mu^2t_2^{2\beta+1}}{4(2\beta+1)}+\frac{r\alpha^2(3\beta+1)\mu^{2\beta+3}}{2(\beta+1)(2\beta+3)(2\beta+1)}]$ (39)

The number of deteriorated items over the period $[0,\ t_2]$ is given by

Production in $[0, t_1]$ - Demand in $[\,0,\mu]$ – Demand in $[0, t_2]$

$=\gamma\,\mathrm{D}_0\int_0^{t_1}te^{-rt}\,dt-\ \mathrm{D}_0\int_0^{\mu}te^{-rt}\,dt-\ \ \mathrm{D}_0\mu\int_\mu^{t_2}e^{-rt}\,dt$

$=\gamma\,\mathrm{D}_0\ (\frac{t_1^2}{2}-\frac{rt_1^3}{3})-\ \mathrm{D}_0\,(\frac{\mu^2}{2}-\frac{r\mu^3}{3})-\mathrm{D}_0\,\mu\,[\ (t_2-\mu)-r\,(\frac{t_2^2}{2}-\frac{\mu^2}{2})]$ (40)

Hence the production cost over the period $[0, t_1)]$is given by

$\int_0^{t_1}Kve^{-ru}\,du\ =\int_0^{t_1}\frac{\alpha_1\gamma}{R^{s-1}}\,e^{-ru}$ du

$=\ \alpha_1\gamma\ \ D_0^{1-s}\ \int_0^{t_1}u^{1-s}$ (1-ru) du

$=\ \alpha_1\gamma\ \ D_0^{1-s}\,[\frac{t_1^{2-s}}{(2-s)}-\frac{rt_1^{1-s}}{(1-s)}]$ (41)

\Shortage cost over the period [0, T] is given by

$$\int_{t_2}^{t}\theta(t)dt\ =\ -\int_{t_2}^{T}D_0\,\mu[\{t_2-t)-\delta[T-t_2](t_2-t)]dt\,e^{-rt}$$

$$= -D_0\,\mu\left[\left\{Tt_2-\frac{t_2^2}{2}-\frac{T^2}{2}-\delta\left(\frac{3t_2T^2}{2}-\frac{3t_2^2T}{2}-\frac{T^2}{2}+\frac{t_2^3}{2}\right)-r\frac{T^2t_2}{2}+r\frac{t_2^3}{2}+r\frac{T^3}{3}+\delta r\,(T-t_2)\left(\frac{t_2T^2}{2}-\frac{t_2^3}{6}-\frac{T^3}{3}\right)\right\}\right] \quad (42)$$

Lost sale cost per cycle is

LS= $D_0\,\mu\int_{t_2}^{T}(1-e^{-\delta(T-t_2)})\,dt$

LS= $D_0\,\mu\int_{t_2}^{T}\delta(T-t_2)(t_2-t)\,dT$

sale cost over the period [0,T] is

$$= D_0\,\mu\delta\left[\frac{3T^2t_2}{2}-\frac{3t_2^2T}{2}-\frac{T^3}{2}+\frac{t_2^3}{2}-\frac{5rt_2T^3}{6}-\frac{rt_2^4}{6}+\frac{rTt_2^3}{6}+\frac{rT^4}{3}+\frac{rt_2^2T^2}{2}\right] \quad (43)$$

From (39)(40),(41),(42),(43), the total average inventory cost X of the system is Lost

$$X=\frac{1}{t_2}[c_1\{D_0(\gamma-1)[\frac{t_1^3}{6}-\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(\beta+2)(2\beta+3)}-\frac{rt_1^4}{8}+\frac{\alpha\beta rt_1^{\beta+4}}{2(\beta+2)(\beta+4)}+\frac{r\alpha^2(3\beta+2)t_1^{2\beta+4}}{2(2\beta+2)(\beta+2)(2\beta+4)}]+D_0[\frac{\gamma\mu t_1^2}{2}-\frac{\gamma t_1^3}{2}+\frac{\alpha\gamma t_1^{\beta+2}\mu^2}{\beta+2}-\frac{\alpha\gamma\beta t_1^{\beta+3}}{2(\beta+2)(\beta+1)}+\frac{\alpha^2\gamma t_1^{2\beta+2}\mu}{2(2\beta+2)}-\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\mu^3}{6}+\frac{t_1^3}{6}+\frac{\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha^2(3\beta+2)\mu^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)}-\frac{\alpha\gamma\mu^{\beta+1}t_1^2}{2(\beta+1)}-\frac{\alpha^2\gamma\mu^{\beta+1}t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(2\beta+3)(\beta+2)}-\frac{\alpha^2\gamma(\beta-2)t_1^{2\beta+3}}{4(\beta+1)(\beta+2)}-\frac{r\gamma t_1^2\mu^2}{4}+\frac{r\gamma t_1^4}{4}-\frac{r\alpha\gamma t_1^{\beta+2}\mu^2}{2(\beta+2)}-\frac{r\gamma\alpha^2t_1^{2\beta+2}\mu^2}{8(\beta+1)}+\frac{r\gamma\alpha^2(\beta^2-4\beta-4)t_1^{2\beta+4}}{8(\beta+1)(\beta+2)^2}+\frac{r\mu^4}{8}-\frac{r\alpha\beta\mu^{\beta+4}}{2(\beta+2)(\beta+4)}+\frac{r\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)}-\frac{r\alpha^2(3\beta+2)\mu^{2\beta+4}}{8(\beta+1)(\beta+2)^2}+\frac{r\alpha^2(3\beta+2)\,t_1{}^{2\beta+4}}{8(\beta+2)^2(\beta+1)}+\frac{r\alpha\gamma\mu^{\beta+2}t_1^2}{2(\beta+2)}+\frac{r\alpha^2\gamma t_1^{\beta+2}\mu^{\beta+2}}{(\beta+2)^2}]+D_0\mu[\frac{t_2^2}{2}-t_2\,\mu+\frac{\mu^2}{2}-\frac{\alpha\mu t_2^{\beta+1}}{(\beta+1)}+\frac{\alpha\beta t_2^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\mu^{\beta+1}t_2}{(\beta+1)}+\frac{\alpha^2(\beta-1)t_2^{2\beta+2}}{2(2\beta+2)(\beta+1)}-\frac{\alpha^2\mu t_2{}^{2\beta+1}}{2(2\beta+1)}-\frac{\alpha^2(3\beta+1)\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)(\beta+1)}+\frac{\alpha^2\mu^{\beta+1}t_2^{\beta+1}}{(\beta+1)^2}-\frac{rt_2^3}{6}+\frac{r\mu^2t_2}{2}-\frac{r\mu^3}{3}-\frac{\alpha\beta rt_2^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{r\alpha\mu^2t_2^{\beta+1}}{2(\beta+1)}+\frac{\alpha r\beta\mu^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{r\alpha\mu^{\beta+2}t_2}{(\beta+2)}-\frac{r\alpha^2(\beta-2)t_2^{2\beta+3}}{4(\beta+2)(2\beta+3)}+\frac{r\alpha^2\mu^2t_2^{2\beta+1}}{4(2\beta+1)}+\frac{r\alpha^2(3\beta+1)\mu^{2\beta+3}}{2(\beta+1)(2\beta+3)(2\beta+1)}\}+c_3\{\gamma D_0(\frac{t_1^2}{2}-\frac{rt_1^3}{3})-D_0(\frac{\mu^2}{2}-\frac{r\mu^3}{3})-D_0\mu((t_2-\mu)-r\left(\frac{t_2^2}{2}-\frac{\mu^2}{2}\right))\}-c_4D_0\,\mu[\left\{Tt_2-\frac{t_2^2}{2}-\frac{T^2}{2}-\delta\left(\frac{3t_2T^2}{2}-\frac{3t_2^2T}{2}-\frac{T^3}{2}+\frac{t_2^3}{2}\right)-r\frac{T^2t_2}{2}+r\frac{t_2^3}{2}+r\frac{T^3}{3}+\delta r\,(T-t_2)\left(\frac{t_2T^2}{2}-\frac{t_2^3}{6}-\frac{T^3}{3}\right)\right\}+c_5\,D_0\,\mu\delta\{\frac{3T^2t_2}{2}-\frac{3t_2^2T}{2}-\frac{T^3}{2}+\frac{t_2^3}{2}-\frac{5rt_2T^3}{6}-\frac{rt_2^4}{6}+\frac{rTt_2^3}{6}+\frac{rT^4}{3}+\frac{rt_2^2T^2}{2}\}+\alpha_1\gamma\,D_0^{1-s}\{\frac{t_1^{2-s}}{(2-s)}-\frac{rt_1^{1-s}}{(1-s)}\}] \quad (44)$$

Optimum values of t_1 *and* t_2 for minimum average cost are obtained as in Case 1 which gives

$$C_1\left\{(\gamma-1)D_o\left(\frac{t_1^2}{2}-\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}\right)-\frac{\alpha^2(3\beta+2)t_1^{2\beta+2}}{2(2\beta+2)(\beta+2)}-\frac{rt_1^3}{2}+\frac{\alpha\beta r(3\beta+2)t_1^{\beta+3}}{2(\beta+2)}+\frac{r\alpha^2(3\beta+2)t_1^{2\beta+3}}{2(2\beta+2)(\beta+2)}\right)+D_o\left(\gamma\mu t_1-\frac{3}{2}\gamma t_1^2+\alpha\gamma\mu^2t_1^{\beta+1}-\frac{\alpha\gamma\beta(\beta+3)t_1^{\beta+2}}{2(\beta+2)(\beta+1)}+\frac{\alpha^2\gamma t_1^{2\beta+1}\mu}{2}-\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}+\frac{t_1^2}{2}-\frac{\alpha\gamma\mu^{\beta+1}t_1}{(\beta+1)}-\frac{\alpha^2\gamma\mu^{\beta+1}t_1^{\beta+1}}{(\beta+1)}-\frac{\alpha^2(3\beta+2)t_1^{2\beta+2}}{2(2\beta+2)(\beta+2)}-\frac{\alpha^2\gamma(\beta-2)t_1^{2\beta+2}(2\beta+3)}{4(\beta+1)(\beta+2)}-\frac{r\gamma t_1\,\mu^2}{2}+r\gamma t_1^3-\frac{r\alpha\gamma\mu^2t_1^{\beta+1}}{2}-\frac{r\gamma\alpha^2\mu^2(2\beta+2)t_1^{2\beta+1}}{8(\beta+1)}+\frac{r\gamma\alpha^2(\beta^2-4\beta-4)t_1^{2\beta+3}}{4(\beta+1)(\beta+2)}-\frac{rt_1^2}{2}+\frac{r\alpha\beta t_1^{\beta+3}}{2(\beta+2)}+\frac{r\alpha^2(3\beta+2)t_1^{2\beta+3}}{4(\beta+2)(\beta+1)}+\frac{r\alpha\gamma\mu^{\beta+2}t_1}{(\beta+2)}+\frac{r\alpha^2\gamma t_1^{\beta+1}\mu^{\beta+2}}{(\beta+2)}\right\}+C_3\gamma D_o(t_1-rt_1^2)+\alpha_1\gamma D_o^{1-s}(t_1^{1-s}-rt_1-s)=0 \quad (45)$$

and

$$C_1D_0\mu\left(t_2-\mu-\alpha\mu t_2^{\beta}+\frac{\alpha\beta t_2^{\beta+1}}{(\beta+1)}+\frac{\alpha\mu^{\beta+1}}{(\beta+1)}+\frac{\alpha^2(\beta-1)t_2^{2\beta+1}}{2(\beta+1)}-\frac{\alpha^2\mu t_2^{2\beta}}{2}+\frac{\alpha^2\mu^{\beta+1}t_2^{\beta}}{(\beta+1)}-\frac{rt_2^2}{2}+\frac{r\mu^2}{2}-\frac{\alpha\beta rt_2^{\beta+2}}{2(\beta+2)}+\frac{r\alpha\mu^2t_2^{\beta}}{2}-\frac{r\alpha\mu^{\beta+2}}{(\beta+2)}-\frac{r\alpha^2(\beta-2)t_2^{2\beta+2}}{4(\beta+2)}+\frac{r\alpha^2\mu^2t_2^{2\beta}}{4}\right)-C_3D_0\mu(1-rt_2)-C_4\,D_0\mu\left\{(T-t_2)-\delta\left(\frac{3T^2}{2}-3t_2T+\frac{3t_2^2}{2}\right)-\frac{rT^2}{2}+\frac{3}{2}rt_2^2+\frac{2}{3}\delta rt_2^3-\delta rt_2T^2+\frac{5}{6}\delta rT^3-\frac{1}{2}\delta rTt_2^2\right\}+C_5D_0\mu\delta\left(\frac{3}{2}T^2-3t_2T+\frac{3}{2}t_2^2-\frac{5rT^3}{6}-\frac{2}{3}rt_2^3+\frac{rTt_2^2}{2}+rt_2T^2\right)-X=0 \quad (46)$$

Numerical Examples

Lets us consider the inventory system with following data:

For case I ($\mu \le t_1 \le t_2$)

$D_0 = 15$, $s = 1.6$, $\mu = 2$, $\alpha_1 = 2$, $\beta = 0.06$, $\alpha = 0.08$, $\gamma = 2$, $r = 0.03$, $c_1 = 4$, $C_4 = 18$, $C_5 = 1.2$, $C_3 = 2.5$, $\delta = 0.5$ *and* $T = 5$

output results are

$t_1 = 2.20206$, $t_2 = 3.88174$, $T.C = 133.177$

Graphical representation of the converities of t_1 and t_2 w.r.t. T.C for case 1. ($\mu \le t_1 \le t_2$)

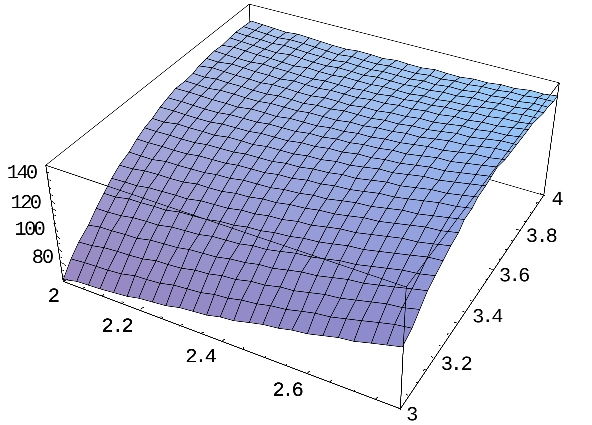

Figure 1. Convexity of t_1 and t_2 $w.r.t$ $T.C$

Table 1. Sensitivity Analysis: The Sensitivity analysis of the key parameter s, r, α, D_0 are given in the below Table for case I.

Parameters		t_1	t_2	**T.C**
r	0.031 0.032 0.033 0.034	2.21034 2.21401 2.21773 2.22149	3.87322 3.87539 3.87758 3.87979	138.003 139.89 141.777 143.665
α	0.09 0.10 0.11 0.12	2.24434 2.35915 2.28212 2.32042	3.8.4784 3.79136 3.82882 3.81021	133.39 125.091 130.644 127.878
D_0	16 17 18 19	2.20702 2.2069 2.20679 2.2067	3.86844 3.86943 3.8703 3.87107	144.888 153.663 162.442 171.223
S	1.3 1.4 1.5 1.6	2.20762 2.20716 2.20658 2.20605	3.87302 3.87604 3.87786 3.879	134.531 133.797 133.4 133.176

The following points are observed

1. t_1 & t_2 increase and T.C also increase with the increase in value of the parameter r
2. t_1 increases while t_2 & T.C decrease with the increase in value of the parameter α.
3. t_1 decreases while t_2 & T.C increase with the increase in value of the parameter Do.
4. t_1 & T.C decrease while t_2 increases with the increase in value of the parameter s.

For case 2 ($t_1 \le \mu \le t_2$)

T = 3, s = 1.3, μ = 0.9, α_1 = 2.2, β = 0.04, α = 0.08, γ = 2.2, r = 0.02, D_0 = 14, C_1 = 2, C_4 = 4, C_5 = 0.4, C_3 = 2.4, δ = 0.5,

Output results are

t_1= 0.0158244, t_2 =2.666, T.C=94.5816

Conclusion

In this study, an EOQ model with ramp type demand rate and unit production cost under inflationary condition has been developed. The quality and quantity of goods decrease in course of time due to deterioration it is a natural phenomena .Hence consideration of Weibull distribution time varying deterioration function defines a significant meaning of perishable, volatile and failure of any kind of item. Shortages are allowed and partially backlogged. The another considered phenomenon viz inflation plays an important role in realistic scenario. A mathematical model has been developed to determine the optimal ordering policy cost which minimizes the present worth of total optimal cost. Thus the model concludes with numerical examples.

Equation (24) and (25) are non- linear equation in t_1 and t_2. These simultaneous non-linear equations can be solved for suitable choice of the parameters c_1, c_3, c_4, c_5, $\alpha, \beta, r, \gamma, \mu, \delta$, D_0, α_1 and s (≠2). If t_1^* and t_2^* are the solution of (24) and (25) for Case I, the corresponding minimum cost $c^*(t_1, t_2)$ can be obtained from (23). It is very difficult to show analytically whether the cost function C (t_1, t_2) is convex. That is why, C (t_1, t_2) may not be global minimum. If C (t_1, t_2) is not convex, then C (t_1, t_2) will be local minimum.

Similarly, solution of equations (45) and (46) for Case II can be obtained corresponding minimum cost C (t_1, t_2) can be obtained from (44).

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